Aircraft Trajectories for Reduced Noise Impact

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A method has been developed for studying the feasibility of reducing annoyance due to aircraft noise by modifying the flight trajectories over a community. Numerical optimization is used to compute the optimum flight paths, based upon a parametric form of the trajectories that implicitly includes some of the problem restrictions. The other constraints are formulated as penalties in the performance measure that is to be minimized. Various aircraft on multiple trajectories (landing and takeoff) can be considered. It is found that significant reductions in community annoyance are possible through this approach. A reduction in the amount of searching for the absolute minimum of the performance measure has been achieved through the use of the known statistics of lateral dispersion in the flight paths. In addition, the population distribution is found not to have a dominant influence in determining the optimum set of flight paths when the mixture of aircraft on those paths changes; the optimum set may have to be recomputed for a change in the flight mix.

Introduction

A CONTINUING problem for the airline industry and airport managers is that of noise due to aircraft operations over populated areas. Possible solutions generally fall into one of three categories: noise source modifications (e.g., retrofitting engine nacelles with sound absorption material), land use regulations (e.g., zoning land near an airport as nonresidential), and operational controls (e.g., power cutbacks during takeoff). Of the three, the last is least expensive, but historically has been considered impractical: significant decreases in noise impact required increased pilot work loads and operational modifications that were perceived as decreasing flight safety. Cook et al.[1] have recently demonstrated a method of altering landing trajectories so as to decrease the community annoyance due to noise. This method allowed for the computation of flight paths that were manageable by the pilot and required only standard flight maneuvers.

We report here some important refinements of this method, including the additions of flight profile optimization, takeoff trajectory optimization, and the ability to handle large airports (~ 15 nominal flight paths, with 500 operations per day). The results show that a sizable decrease in noise impact is possible and that the optimum set of trajectories depends not only on the population distribution, but on the types and numbers of aircraft operations over the community as well.

Description of the Method

In order to determine aircraft trajectories that are improved with respect to noise effects, it is necessary to develop operational models of population distribution, aircraft noise propagation, annoyance due to noise, and aircraft flight paths. Additionally, some form of optimization scheme must be chosen and implemented, along with a cost function (performance measure) based upon the collection of models. The models described here are intended to facilitate computation and demonstrate the feasibility of the general method. Because of the modular approach used in their implementation, it is possible to replace any of them with a more refined version.

Population Model

The population distribution model used is the same as that of Cook et al.[1] It consists of a discrete distribution of people (surrounding a given airport), calculated by overlaying a map of the area with a grid of simple geometric shapes, and then determining the number of people in each grid section via U.S. Census data. The distribution of people within a section is assumed to be uniform. For a complete description, see Ref. 1.

Aircraft Noise Model

Each aircraft is assumed to behave as an isotropic point source of noise. As a result, the A-weighted sound level L_A at a given slant range r from an aircraft operating with given power settings may be approximated as

$$L_A = C_1 - C_2 \times \log_{10} r \tag{1}$$

where C_1 and C_2 are constants for each aircraft. Typical values of C_1 and C_2 for some commercial aircraft are shown in Table 1.

Annoyance Model

Two of the annoyance measures considered require the calculation of the average day-night sound level L_{dn} which is based upon the weighted average sound power density (sometimes referred to as "energy averaging") over a 24-h period. The relation between L_A and L_{dn} is given by[2]

$$L_{dn} = 10 \times \log_{10} \left\{ \sum_{i=1}^{N} w_i 10^{L_{A,i}/10} \Big/ N \right\} \tag{2}$$

where $L_{A,i}$ is the A-weighted sound level at the ith sample time; N is the number of samples in 24 h; and w_i is the time of day factor:

$$w_i = 1 \text{ from 7 a.m. to 10 p.m.}$$

$$= 10 \text{ from 10 p.m. to 7 a.m.}$$

Any attempt to quantify the annoyance from noise must necessarily include both the sound level intensities and the numbers of people affected. The single measure of annoyance adopted by the National Academy of Science is the Noise Impact Index (NII), described in Ref. 3. Calculation of NII is

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Table 1 L_A vs slant range for some commercial aircraft[a]

Aircraft	C_1	C_2
DC-8-30	155.74	28.32
DC-9 w/SAM	142.28	24.65
DC-10-10	146.22	28.80
707 w/SAM	135.11	23.60
720	140.88	21.95
727-200	139.47	22.66
727 w/SAM	124.28	18.74
737-100/200	154.74	29.52
737 w/SAM	147.31	26.96
747-200	139.89	24.78
L-1011	136.94	24.49
A-300	166.47	37.20
BAC-111	150.72	27.06
VC-10	142.65	22.95
CV-990	157.34	27.16

[a]Data obtained from the Integrated Noise Model Data Base, Federal Aviation Administration, version 2, released July 31, 1979; data reduced by least-squares-error fits.

based upon another measure, Level-Weighted Population (LWP):

$$\text{LWP} = \sum_{L_{dn}} P(L_{dn}) W(L_{dn}) \tag{3}$$

$$\text{NII} = \text{LWP} \Big/ \sum_{L_{dn}} P(L_{dn}) \tag{4}$$

where $P(L_{dn})$ is the number of people experiencing an average day-night level L_{dn}, and $W(L_{dn})$ is a weighting factor for that L_{dn}, based on studies by Schultz.[4]

Restrictions

The restrictions that pertain directly to the feasibility of the trajectories can be grouped into five categories: aircraft dynamics, passenger comfort, statutory regulations, piloting considerations, and the maximum allowable number of high-level noise events for a given population grid cell.

Aircraft Dynamics Constraint

The aircraft dynamics constraints limit a trajectory to one which given types of aircraft are capable of flying. The restrictions may be stated in terms of fixed properties of the aircraft and maximum allowable control surface deflections. In the event of several restrictions on any one quantity, the most severe prevails. The following expressions, developed from linear equations of aircraft motion[5,6] are divided into restrictions on the lateral and longitudinal motions. Refer to Figs. 1 and 2.

1) Lateral constraint:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \Big/ \frac{d^2y}{dx^2} = R(x) \geq \bar{V} / [C \times \min(\delta r_1, \delta r_2, \delta r_3)] \tag{5}$$

where $y(x)$ is the ground track of the trajectory; $R(x)$ is the radius of curvature; $\bar{V}$ is the average total velocity; the δr_i depend on the maximum rudder and aileron deflections; and C depends on the stability derivatives of the aircraft.

2) Longitudinal constraint:

$$\gamma_{d,\max} \leq \gamma \leq \gamma_{c,\max} \tag{6}$$

where γ is the flight-path angle with respect to the horizontal, and $\gamma_{d,\max}$ and $\gamma_{c,\max}$ are the maximum allowable angles of descent and climb, respectively.

Passenger Ride Quality Constraint

The passenger comfort, or ride quality, constraint limits the operation of an aircraft to those maneuvers found statistically to be acceptable to the passengers. An expression of these limitations on bank angle for an aircraft executing a coordinated turn is[7]

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \Big/ \frac{d^2y}{dx^2} \geq \frac{\bar{V}^2}{C_2 g} \tag{7}$$

where C_2 depends on the percentage of passengers who will find the motion acceptable (e.g., $C_2 = 0.50$ corresponds to 80% of the passengers rating the maneuver "satisfactory," and $C_2 = 0.19$ corresponds to 90%); $\bar{V}$ is the average total velocity during the turn; and g is the acceleration due to gravity.

Regulatory Constraints

Regulations affecting allowable flight paths may pertain either to safety, governmental security, or noise annoyance. Safety restrictions involve single trajectories (e.g., avoidance of low-altitude obstacles) and multiple paths (e.g., maintenance of safe separating distances); military installations often prohibit general and commercial aviation flights from passing too closely for security reasons; and there may be certain areas (e.g., schools and hospitals) which are particularly sensitive to noise from nearby aircraft.

Two safety restrictions that apply to every airport have been included in this study. They are

1) A safe distance should be maintained between any two trajectories as far as is possible. Because several trajectories may share a runway, the region in which this restriction applies often excludes an area within several miles of the airport. The minimum allowed separating distance used in this work is 800 m.

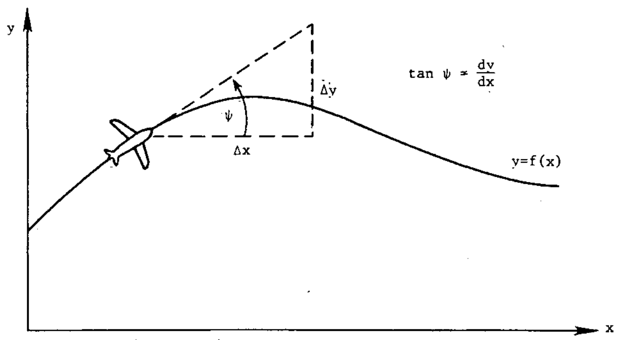

Fig. 1 Horizontal flight-path angle.

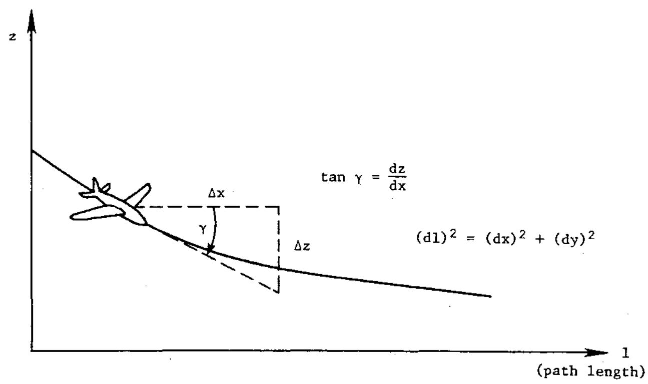

Fig. 2 Vertical flight-path angle.

2) No landing aircraft should be required to execute a turn of more than 45 deg, as it crosses the inner marker, in order to come into alignment with the runway.

Pilot Operating Constraints

It is possible that the optimum trajectories computed would require more maneuvering of an aircraft than a pilot can or is willing to perform, especially in the vicinity of the airport, where his attention is required on a number of matters. The constraints then are to limit the number and complexity of the turning maneuvers on the flight paths.

Threshold Noise Constraint

A difficulty which must be avoided is the possibility of modifying the trajectories such that the overall annoyance of the community is reduced, but at the expense of raising the noise levels significantly for a section of the populace. To avoid this, a constraint is imposed that prohibits any area (grid cell) from receiving maximum noise levels ($L_{A,\max}$) during a flyover, at or above a "threshold" level, more than $N_{\max}$ times per day. The threshold level and value of $N_{\max}$ should be made as small as possible, without overly restricting the possible trajectories.

Flight-Path Model

In selecting an appropriate parametric form for specifying a trajectory, it is necessary to weigh the benefits of a large number of parameters—more degrees of freedom and, hence, more flexibility—against the attendant increase in computation needed to determine the values of those parameters. Chang,[8] for example, reported that using only five terms in a Fourier series representation of aircraft landing trajectories (the unknown parameters were the Fourier coefficients) led to overly complicated flight paths, while increasing the number of terms would have resulted in unacceptable computational requirements without necessarily simplifying the paths.

The model chosen for a flight path in this work avoids that problem by directly relating the maximum number of turning maneuvers to the number of parameters to be determined. Each trajectory is represented by a chain of linear segments joined by helical arcs (circular in the ground plane, linear in the profile). Figures 3 and 4 show an example of a path with three segments. The two intersections of the linear segments in Fig. 3 are called "corner points"; the Cartesian coordinates of these points, along with those of the other trajectories, make up the set of parameters to be determined. In the ground plane, a circular arc with minimum radius (determined by the dynamic and passenger comfort constraints) is inscribed in the angle between any two linear segments. The arc, tangent to both segments, describes the ground track of the path flown during the turn at the corner. Between tangent points, as shown in Fig. 4, the profile of the path is approximated by a linear segment. The resulting turn, in three dimensions, is a helical arc. Since each corner point (requiring three coordinates) may have at most one turn, the complexity of the flight path is limited in a natural way by the allowed flexibility, i.e., the number of corner points used.

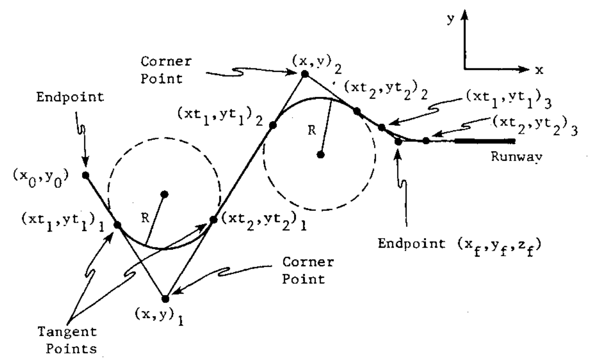

Fig. 3 Ground track, linear segment representation.

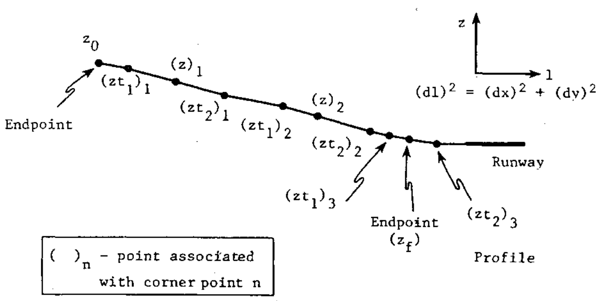

Fig. 4 Profile, linear segment representation.

Optimization Scheme

The iterative descent method of Fletcher et al.,[9] modified by Luenberger,[10] has been adopted for computing the optimum trajectory parameters (corner points). This method offers the advantage of improved convergence rates for problems containing penalty functions, which often inhibit the iterative progress of most optimization algorithms.

Cost Function

The essential part of the cost function (performance measure) is the annoyance measure (NII or LWP); however, none of the constraints should be violated in reducing the value of the cost function. These constraints are incorporated into the total cost as quadratic penalty functions of the form

$$P_i = \{\max[0, (g_i(x) - c_i)]\}^2 \tag{8}$$

for each constraint of the form

$$g_i(x) \le c_i \tag{9}$$

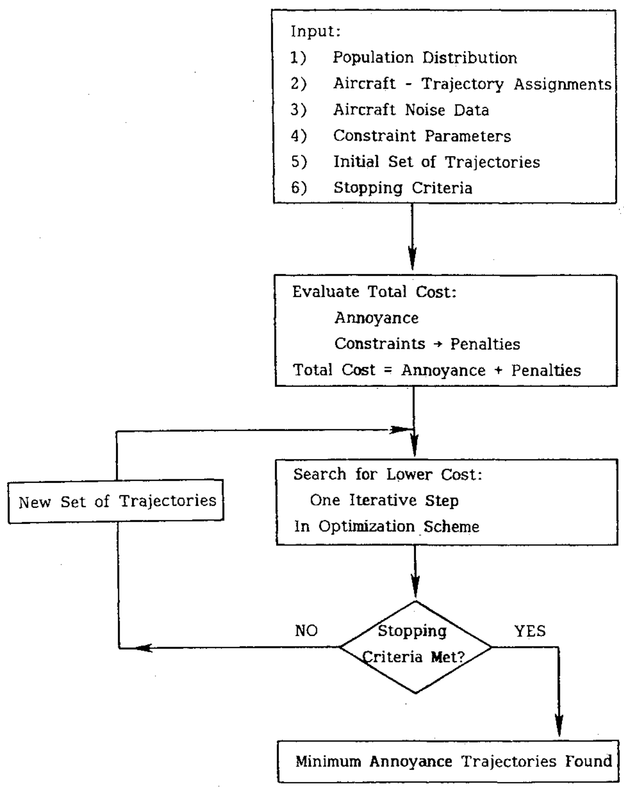

Fig. 5 System flowchart.

where $g_i(x)$ is a constraint function, perhaps implicitly, of x, the vector of unknown parameters, and c_i is the maximum allowable value for g_i. The total cost function is then

$$f(x) = \mu_A A(x) + \Sigma \mu_i P_i(x) \tag{10}$$

where $A(x)$ is the annoyance measure and the μ_i are adjustable weighting factors.

The Integrated System

Combining the models of annoyance, flight path, and cost function with the optimization scheme results in a system whose operation is summarized by the flowchart in Fig. 5. Some special considerations must be given to the problem of local minima in the cost function, and to the case of large airports.

The customary procedure for insuring that the iterative search has found the global minimum of the cost function is to perform the search from different starting points (in this case, different initial sets of trajectories). A scheme for systematically choosing these starting points involves the statistics of lateral dispersion of individual flights from their specified trajectories. Based on studies by Galloway,[11] the angular dispersion of landing paths for commercial aircraft is 3 deg; for takeoff paths headed straight-out, 9 deg, with some additional dispersion added during turns. Figure 6 illustrates the dispersion occurring in the 15 nominal flight paths at Lambert-St. Louis International Airport.

Searching for the optimum set of trajectories within the dispersion limits (shaded regions) of Fig. 6 is unprofitable, since such an optimum is not significantly different in a statistical sense from the nominal set. Different starting points (representing significantly different trajectory sets) are used to perform the optimization a number of times. The lowest valued local minimum is then taken to be the global minimum.

Choices for the starting point are limited by two factors: the lateral dispersion on these paths and the constraints (particularly the aircraft dynamic and passenger comfort restrictions). For example, as shown in Fig. 7, there are only five choices for track 7 starting points. Moving track 7 farther to the east would cause it to overlap track 8, in which case the two paths would be replaced by one. Consideration of cases farther to the north involves the same result with track 3. Thus the presence of many trajectories limits the required amount of global searching, an important saving, since the amount of computation needed for the optimization increases with the number of flight paths. In situations where the search has converged to a local minimum not significantly different from the starting point (but the existence of a different minimum is suspected), restarting the optimization after perturbing the corner points by some amount (~1000 m) may free the search from the region of the local minimum.

Results

Two computer programs for implementing the system described have been developed. The first, MANIP2, is a revision of the program described in Ref. 1, with the following refinements: 1) optimization of a flight path's vertical profile is included; 2) takeoff and landing trajectories may be optimized, simultaneously; and 3) a larger variety of commercial aircraft noise data is included. The second program, CYCLIC, has the same features as MANIP2, except that 1) it employs the "equivalent aircraft" concept‡ for computing aircraft noise levels, and 2) the optimization scheme is modified to consider only a few trajectories at a time in order to reduce storage requirements for large airports (i.e., many trajectories). Both programs were written in FORTRAN IV for use on CDC Cyber 170 series machines.

‡All of the aircraft on a given flight path are replaced by a single source that delivers an equivalent amount of round energy (per 24-h period) to each population cell. For details, see Ref. 12 or 13.

The results of several investigations appear in this section. Two airports were used in these studies: Phoenix Sky Harbor International and Lambert-St. Louis International. Their respective population distributions are depicted in Figs. 8 and 9.

Takeoff Trajectories

Using the Lambert-St. Louis Airport community as a test case, a single trajectory was optimized, with LWP as the annoyance criterion. Both the nominal and optimum paths are shown in Fig. 10. The nominal path was approximated by a five-segment trajectory, with a 5-deg flight-path angle (γ); the optimum path was restricted to three segments and $\gamma_{c,\max} = 6$ deg. Nearly all of the variation between the two paths is in the ground tracks. This phenomenon, seen throughout the results that follow, occurs because 1) variations in the population density distribution occur only in the ground plane, and 2) the average distances from the flight paths to the population centers change more rapidly due to horizontal variations in the flight paths than due to vertical variations (owing to the small angles between the ground and the trajectories). The exception is, of course, when a segment near the runway (and closer to the ground) passes close to a population center. Then an increase in the flight-path angle will produce a rapid decrease in the noise level at that center.

A point approximately 90 km east of the airport was chosen as the outermost endpoint. This allowed the position of the corner point 2 to vary without being restricted by the constraints. The optimum trajectory produced a LWP of 1267.9, while the nominal path caused a LWP of 2571.1, a decrease of 50.6%.

Landing Trajectories

An example of the results for optimization of landing paths only is shown in Figs. 11 and 12. There are two entry points (initial approach fixes). The figures show only the ground tracks of the trajectories; the glide slopes are 3 deg. Because the nominal paths are joined together over a large portion of their lengths, it is assumed that separation for safety purposes is not a requirement for traffic control here; therefore the separation constraint is bypassed. Note that there are three segments on each path, with some overlapping.

Using LWP as the annoyance measure, the optimum pair is as shown in Fig. 12. The LWP has decreased from 30,566 to 25,192, an improvement of 17%.

Simultaneous Optimization of Takeoff and Landing Paths

At large airports such as Lambert-St. Louis International, the large variety of aircraft that must be considered necessitates the use of the equivalent aircraft concept in order to keep the computation time manageable. Further, the number of trajectories being optimized (15 at St. Louis) will require a considerable increase in computer storage requirements, compared with those needed for Phoenix. If the storage required is in excess of the allocated computer resources, the dilemma may be remedied by optimizing the flight paths one at a time, cyclically. The sound power density contributions from the (temporarily) constant trajectories are stored and then summed with those of the path being optimized in order to determine the L_{dn} value in each grid cell. As a result, the optimization storage requirements are for only one trajectory, but the annoyance measure is for the entire set of paths.

It should be noted that this approach does not guarantee convergence to a minimum; however, the results for the St. Louis case are excellent. Figure 13 shows the nominal paths at Lambert-St. Louis International (tracks 1-12: takeoff, 5 deg; tracks 13-15: landing, 3 deg). Each trajectory is approximated by five segments. Judging by the population distribution (Fig. 9), it was decided that paths 1, 2, 7-9, and 13-15 could give the most improvement in the annoyance (LWP). Using the program CYCLIC to cyclically optimize these paths, and allowing a maximum of three segments per trajectory, the

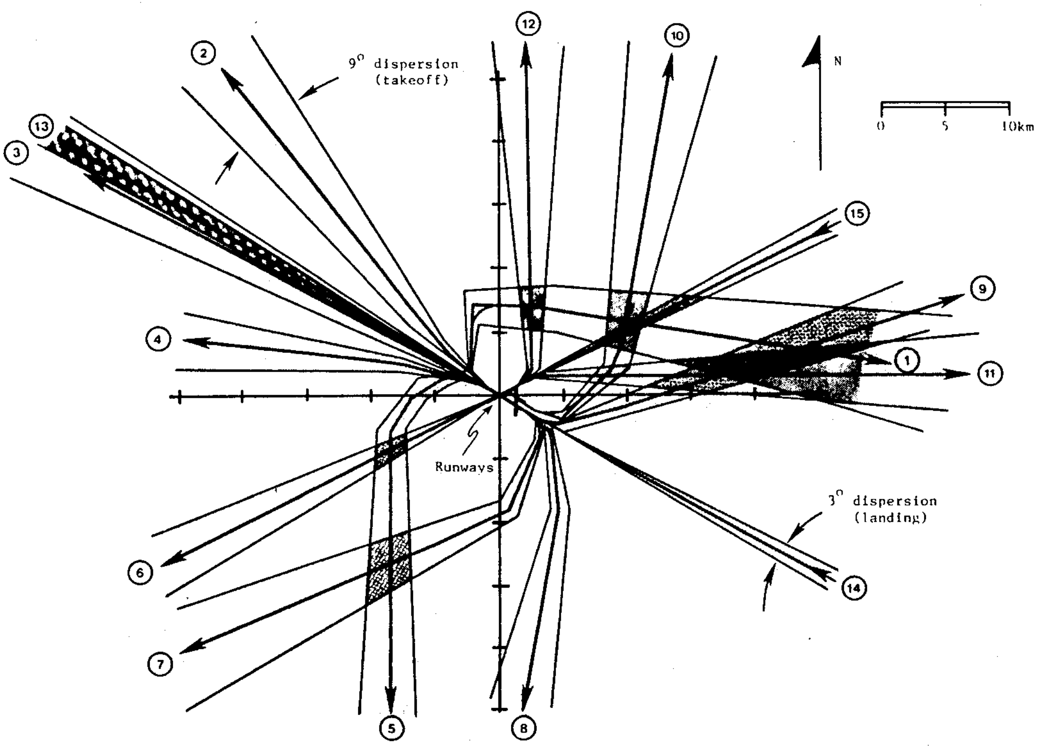

Fig. 6 Lateral dispersion in the nominal flight paths at Lambert-St. Louis International Airport.

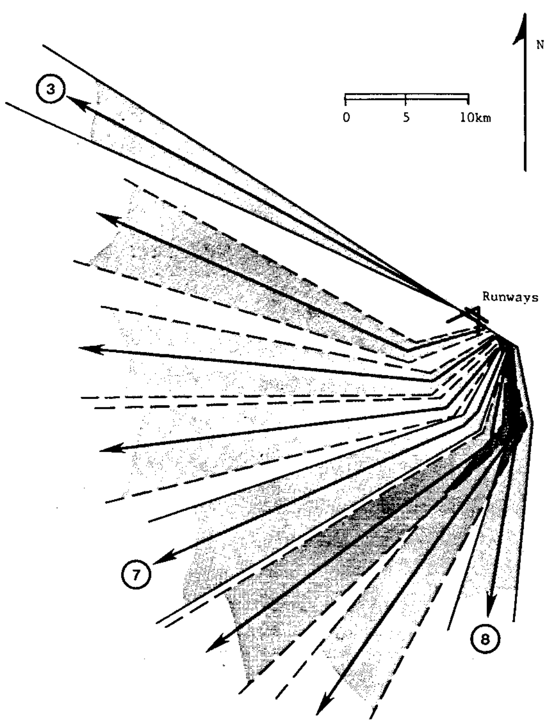

Fig. 7 Starting points for track 7.

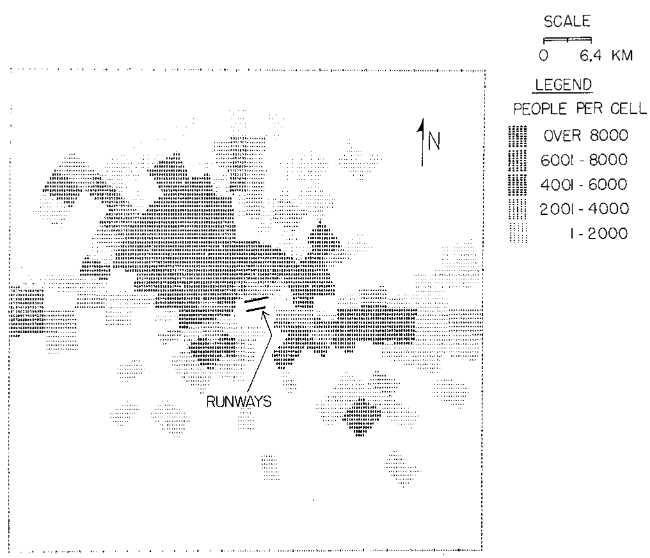

Fig. 8 Phoenix (Sky Harbor International Airport) population distribution.

result was a 26% decrease in LWP. Figure 14 shows the improved trajectories.

Population Distribution and Aircraft Mix

In order to investigate the relative importance of the population distribution vs the aircraft mix as they affect the optimum trajectory set, a study was conducted simulating two different mixtures of aircraft (landings only) at Phoenix. Mix 1 corresponds to the true mix at Phoenix; mix 2 corresponds to a fleet of DC-10 aircraft, with the equivalent amount of passenger seating. The optimum paths for mix 2 are significantly different from those computed using mix 1. Figures 12 and 15 depict the results.

The consequence of this example is that the relative importance of the population distribution vs aircraft mix in determining the optimum trajectories cannot be stated in general. For a simple distribution of people (e.g., clusters of population separated by several miles), "corridors" for optimum trajectories that are independent of aircraft mix may exist. Most distributions, though, are usually more dispersed, which results in more people (on the average) being closer to the flight paths. Because of the nonlinearities in the noise and annoyance models, changes in the aircraft mix result in an unanticipated change in the annoyance. Therefore new optimum flight paths must be computed when the mix changes.

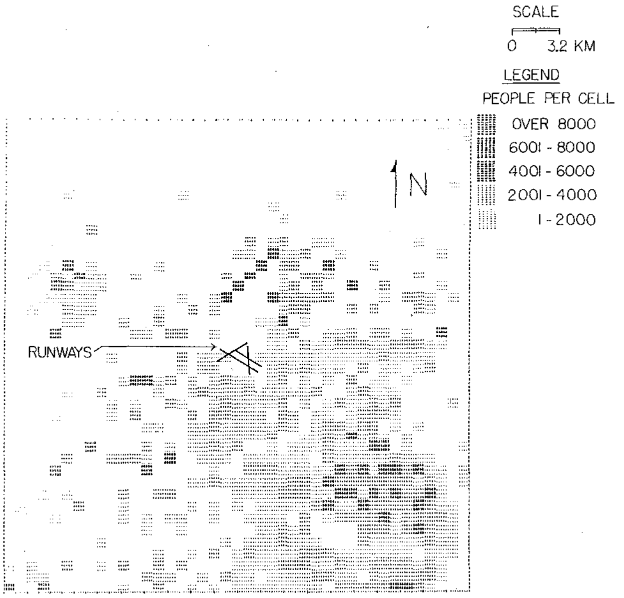

Fig. 9 St. Louis (Lambert-St. Louis International Airport) population distribution.

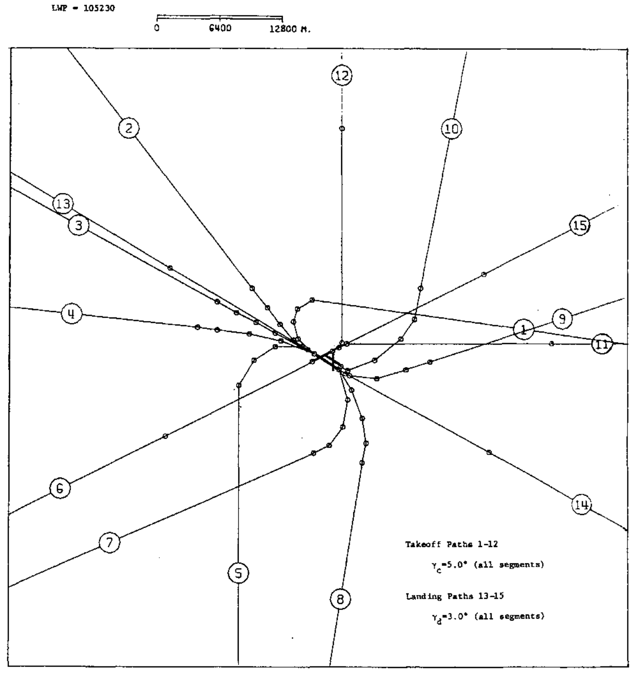

Fig. 13 St. Louis nominal trajectory set.

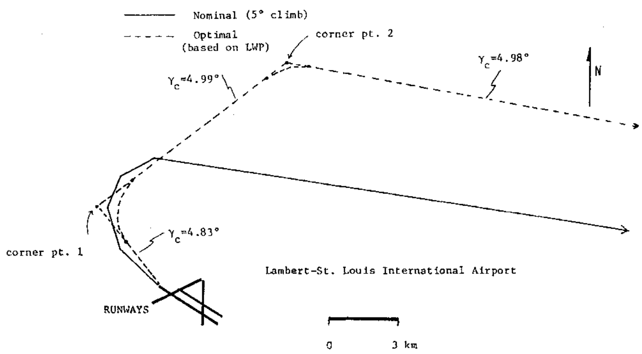

Fig. 10 Single takeoff trajectory—nominal and optimum.

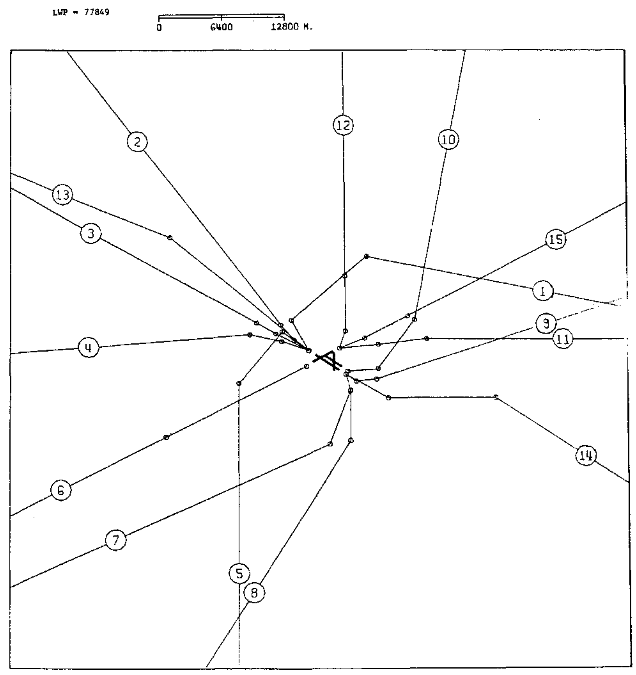

Fig. 14 St. Louis optimum trajectory set.

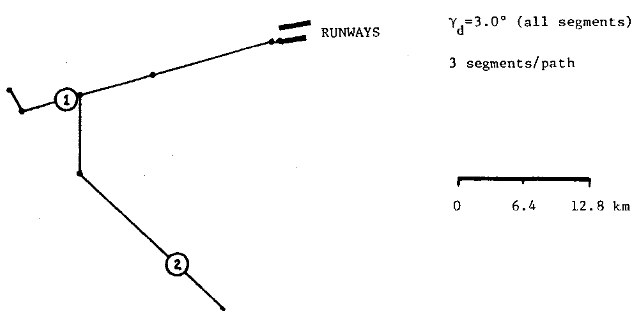

Fig. 11 Nominal trajectories at Phoenix Sky Harbor Airport.

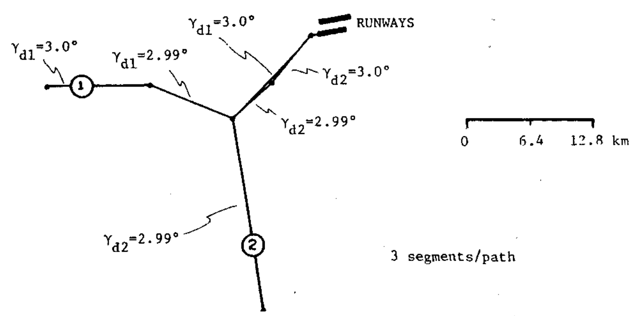

Fig. 12 Optimum (LWP) landing trajectories at Phoenix Sky Harbor Airport.

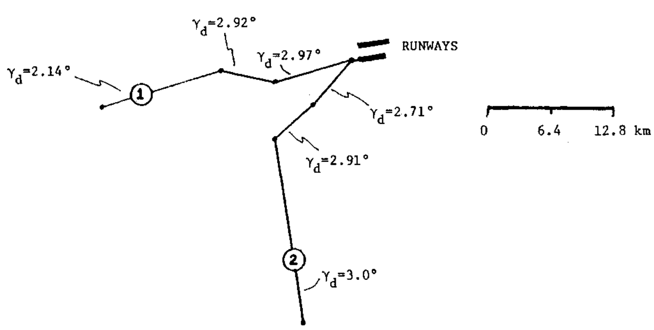

Fig. 15 Optimum (LWP) landing trajectories at Phoenix Sky Harbor Airport (DC-10 mix).

Conclusions

This study has developed a method to assess the ability to reduce, through operational controls, the annoyance caused by aircraft noise in a community. Test cases, using a computer implementation of the method and actual airport situations, have shown the feasibility of using trajectory modifications in the solution to the problem of aircraft noise.

Of particular importance is the finding that the population distribution in a given locality does not necessarily determine a unique set of optimum trajectories; the mixture of aircraft strongly influences the solution. This is unfortunate from a computational aspect, since the mixtures on the flight paths generally will vary over time, requiring that new optimum sets be calculated.

The difficulty associated with searching over many local minima was overcome by making use of the lateral dispersion in the trajectories. These dispersions limit the size of the alternative region (in the horizontal plane) in which to search for significantly different optimum paths.

Acknowledgment

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